**Chapter 10**

**Transportation, Assignment, and Transshipment Problems**

**Solutions:**

1. a. & b.

The linear programming formulation and optimal solution as printed by The Management Scientist are shown below. The first two letters in the variable names identify the “from” node for the shipping route and the last two identify the “to” node. Also, The Management Scientist prints ‘<’ for ‘.’

LINEAR PROGRAMMING PROBLEM

MIN 2PHAT + 6PHDA + 6PHCO + 2PHBO + 1NOAT + 2NODA + 5NOCO + 7NOBO

S.T.

1) PHAT + PHDA + PHCO + PHBO < 5000

2) NOAT + NODA + NOCO + NOBO < 3000

3) PHAT + NOAT = 1400

4) PHDA + NODA = 3200

5) PHCO + NOCO = 2000

6) PHBO + NOBO = 1400

OPTIMAL SOLUTION

Objective Function Value = 24800.000

Variable Value Reduced Costs

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PHAT 1400.000 0.000

PHDA 200.000 0.000

PHCO 2000.000 0.000

PHBO 1400.000 0.000

NOAT 0.000 3.000

NODA 3000.000 0.000

NOCO 0.000 3.000

NOBO 0.000 9.000

Note that the Philadelphia port satisfies all the demand at Atlanta, Columbus, and Boston as well as the portion of the Dallas demand exceeding the New Orleans capacity.

3. a.



b. Let *xij* = amount shipped from supply node *i* to demand node *j*.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Min | 10*x*11 | + | 20*x*12 | + | 15*x*13 | + | 12*x*21 | + | 15*x*22 | + | 18*x*23 |  |  |
| s.t. |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | *x*11 | + | *x*12 | + | *x*13 |  |  |  |  |  |  |  | 500 |
|  |  |  |  |  |  |  | *x*21 | + | *x*22 | + | *x*23 |  | 400 |
|  | *x*11 |  |  |  |  | + | *x*21 |  |  |  |  | = | 400 |
|  |  |  | *x*12 |  |  |  |  | + | *x*22 |  |  | = | 200 |
|  |  |  |  |  | *x*13 |  |  |  |  | + | *x*23 | = | 300 |

*xij*  0 for all *i*, *j*

c. Optimal Solution

|  |  |  |
| --- | --- | --- |
|  | Amount | Cost |
| Southern - Hamilton | 200 | $ 2000 |
| Southern - Clermont | 300 | 4500 |
| Northwest - Hamilton | 200 | 2400 |
| Northwest - Butler | 200 | 3000 |
| Total Cost |  | $11,900 |

d. To answer this question the simplest approach is to increase the Butler County demand to 300 and to increase the supply by 100 at both Southern Gas and Northwest Gas. The new optimal solution is:

|  |  |  |
| --- | --- | --- |
|  | Amount | Cost |
| Southern - Hamilton | 300 | $ 3000 |
| Southern - Clermont | 300 | 4500 |
| Northwest - Hamilton | 100 | 1200 |
| Northwest - Butler | 300 | 4500 |
| Total Cost |  | $13,200 |

From the new solution we see that Tri-County should contract with Southern Gas for the additional 100 units.

5. a.



b. Let *xij* = number of hours from consultant *i* assigned to client *j*.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Max | 100*x*11 | + | 125*x*12 | + | 115*x*13 | + | 100*x*14 | + | 120*x*21 | + | 135*x*22 | + | 115*x*23 |  |  |  |  |
| s.t. |  | + | 120*x*24 | + | 155*x*31 | + | 150*x*32 | + | 140*x*33 | + | 130*x*34 |  |  |  |  |  |  |
|  | *x*11 | + | *x*12 | + | *x*13 | + | *x*14 |  |  |  |  |  |  |  |  |  | 160 |
|  |  |  |  |  | *x*21 | + | *x*22 | + | *x*23 | + | *x*24 |  |  |  |  |  | 160 |
|  |  |  |  |  |  |  |  |  | *x*31 | + | *x*32 | + | *x*33 | + | *x*34 |  | 140 |
|  | *x*11 |  |  | + | *x*21 |  |  | + | *x*31 |  |  |  |  |  |  | = | 180 |
|  |  |  | *x*12 |  |  | + | *x*22 |  |  | + | *x*32 |  |  |  |  | = | 75 |
|  |  |  |  |  | *x*13 |  |  | + | *x*23 |  |  | + | *x*33 |  |  | = | 100 |
|  |  |  |  |  |  |  | *x*14 |  |  | + | *x*24 |  |  | + | *x*34 | = | 85 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

*xij*  0 for all *i*, *j*

Optimal Solution

|  |  |  |
| --- | --- | --- |
|  | Hours Assigned | Billing |
| Avery - Client B | 40 | $ 5,000 |
| Avery - Client C | 100 | 11,500 |
| Baker - Client A | 40 | 4,800 |
| Baker - Client B | 35 | 4,725 |
| Baker - Client D | 85 | 10,200 |
| Campbell - Client A | 140 | 21,700 |
| Total Billing |  | $57,925 |

c. New Optimal Solution

|  |  |  |
| --- | --- | --- |
|  | Hours Assigned | Billing |
| Avery - Client A | 40 | $ 4,000 |
| Avery - Client C | 100 | 11,500 |
| Baker - Client B | 75 | 10,125 |
| Baker - Client D | 85 | 10,200 |
| Campbell - Client A | 140 | 21,700 |
| Total Billing |  | $57,525 |